Corrections - Volume III

- **Page v.** The second line in the second paragraph should read: "Measure, Integration and Martingales"..
- **Page 96.** The second line from the bottom should read: $f : \Omega \to \mathbb{R}$.
- **Page 313.** The second line in Corollary 16.25 should read: $\frac{-dz+b}{cz-a}$.
- **Page 342.** Equation (18.21) should read

$$\cos z = \frac{e^{iz} + e^{-iz}}{2}.$$

- **Page 380.** The line below the figure should read: $\int_\gamma z \, \mathrm{d}z = 0$.
- **Page 406.** Problem 9 a) should read:

$$\int_{|\zeta - z|=1} \frac{\cos \zeta^2}{(\zeta - \sqrt{\pi})^3} \, \mathrm{d}\zeta;$$

- **Page 462.** The third line of Theorem 24.17 should read: ... such that on U we ...
- **Page 462.** The beginning of the Proof of Theorem 24.17 should read:

  Proof. Let $z_0 \in D$. If z_0 is not a singularity of f we can find a neighbourhood U of z_0 such that $f|_U$ is holomorphic. Now choose in U as g the function f and as h the constant function $h(z) = 1$. In the case...

- **Page 463.** The third line from the bottom should read: for $k < -n$
- **Page 469.** Problem 2 c) should read:

$$\frac{\cos 2z}{(z - \frac{\pi}{4})^3} \quad \text{at} \quad z_0 = \frac{\pi}{4}$$

- **Page 640.** (∗∗) in the solution to Problem 11 should read:

$$(**) \qquad \leq \left(\int_{[a,b]} \left(\int_{[a,b]} |k(x,y)|^2 \, \mathrm{d}x \right) \mathrm{d}y \right)^{\frac{1}{2}} \left(\int_{[a,b]} |u(y)|^2 \, \mathrm{d}y \right)^{\frac{1}{2}}$$

- **Page 698.** Solution of Problem 9 a) should read:

  With $f(z) = \cos z^2$ the Cauchy integral formula for $n = 2$ reads

$$f^{(2)}(\sqrt{\pi}) = \frac{2!}{2\pi i} \int_{|\zeta-2|=1} \frac{f(\zeta)}{(\zeta - \sqrt{\pi})^3} \mathrm{d}\zeta = \frac{1}{\pi i} \int_{|\zeta-2|=1} \frac{\cos^2 \zeta}{(\zeta - \sqrt{\pi})^3} \mathrm{d}\zeta$$

  where we used that $1 < \sqrt{\pi} < 2$, i.e. $\sqrt{\pi} \in B_1(2)$. Since $\frac{\mathrm{d}^2}{\mathrm{d}\zeta^2}(\cos \zeta^2) = -2 \sin \zeta^2 - 4\zeta^2 \cos \zeta^2$ it follows that

$$\int_{|\zeta-2|=1} \frac{\cos \zeta^2}{(\zeta - \sqrt{\pi})^3} \mathrm{d}\zeta = \pi i (4\pi^2) = 4\pi^2 i.$$

- **Page 709.** The second line of the solution to Problem 2 b) should read:

$$\begin{aligned}(z - 4) \sin \frac{1}{z+3} &= (w - 7) \sin \frac{1}{w} \\ &= (w - 7) \left(\frac{1}{w} - \frac{1}{3! w^3} + \frac{1}{5! w^5} \pm \cdots \right)\end{aligned}$$

- **Page 712.** The solution to Problem 4 a) should read:

  We have to look at the zeroes of $z \mapsto 4 \sin z - 2$, i.e. we have to solve the equation $\sin z = \frac{1}{2}$. For z real we obtain the points $\frac{\pi}{4} + 2k\pi$ and $\frac{5\pi}{4} + 2k\pi$, $k \in \mathbb{Z}$, and since $\sin' = \cos$ these are simple zeroes. Consequently f has at the points $\frac{\pi}{4} + 2k\pi$ and $\frac{5\pi}{4} + 2k\pi$, $k \in \mathbb{Z}$, a pole of order 2. Using the representation $\sin z = \frac{1}{2i}(e^{iz} - e^{-iz})$ we first deduce that $\sin z = \frac{1}{2}$ cannot have a purely imaginary solution iy. In the general case, i.e. $z = x + iy$, we have to solve

$$\frac{1}{2} = \sin z = \left(\frac{e^{-y} + e^{y}}{2} \right) \sin x + \left(\frac{e^{-y} - e^{y}}{2} \right) i \cos x$$

  where we used $\sin z = \frac{e^{iz} - e^{-iz}}{2i}$. Thus we must have $\left(\frac{e^{-y} - e^{y}}{2} \right) \cos x = 0$ and $(e^{-y} + e^{y}) \sin x = 1$. If $y = 0$ the the first equality holds and the second becomes $\sin x = \frac{1}{2}$ and we are back in the first case discussed. If $y \neq 0$ then we must have $\cos x = 0$ which implies $\sin x \in \{1, -1\}$, but for all $y \in \mathbb{R}$ we have $e^{-y} + e^{y} > 1$ and the second equation cannot hold. Thus the only zeroes of $z \mapsto 4 \sin z - 2$ are those determined in the first case and hence at these points f has poles or order 2.